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Gaugino condensation in an improved heterotic M-theory

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Abstract: Gaugino condensation is discussed in the context of a consistent new version of low energy heterotic M-theory. The four dimensional reduction of the theory is described, based on simple boson and fermion backgrounds. This is generalised to include gaugino condenstates and various background fluxes, some with non-trivial topology. It is found that condensate and quantised flux contributions to the four-dimensional superpotential contain no corrections due to the warping of the higher dimensional metric.

Keywords: Flux compactifications, Superstrings and Heterotic Strings.

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1. Introduction

Horava and Witten [1, 2] proposed some time ago that the low energy limit of strongly coupled heterotic string theory could be formulated as eleven dimensional supergravity on a manifold with boundary. Although the theory has received less attention recently than type IIB superstring theory, it nevertheless possesses advantages over other string theories as a starting point for particle phenomenology [3, 4].

The original formulation of Horava and Witten contained some serious flaws which where corrected recently using a new formulation of supergravity on manifolds with boundary [5 − 7]. The most serious problem affecting the model was that it was expressed as a series in factor ${\kappa_{11}}^{2/3}$ multiplying the matter action, which worked well at leading and next-to-leading order but became ill-defined thereafter. This problem was resolved by a simple modification to the boundary conditions resulting in a low energy theory which is supersymmetric to all orders in ${\kappa_{11}}^{2/3}$.

It is necessary to revisit important issues such as reduction to four dimensions and moduli stabilisation, where there has been much progress recently [8 − 16], in the light of this new formulation of low-energy heterotic M-theory. In this paper we shall focus on the effects of gaugino condensation, since this is the quantity which is most affected by the new boundary conditions.

In the usual reductions, six of the internal dimensions lie on a Calabi-Yau three-fold and one internal dimension stretches between the two boundaries. Often, five-branes are included which run parallel to the boundaries. The reductions have separation moduli between the boundaries or five branes and Calabi-Yau moduli, which naturally split into two families depending on the type of harmonic form: $(1,1)$ and $(2,1)$. A variety of stabilisation mechanisms have been proposed:

- internal fluxes, which might fix the $(2,1)$ moduli by analogy with the moduli stabilisation used in type IIB string theory [17]
- branes stretching between the boundaries or the five-branes [8, 9]
- gaugino condensation, which gives a potential depending on the Calabi-Yau volume [18, 20, 19, 11]

In addition to the stabilisation mechanisms, there also has to be added some means of ensuring that the effective cosmological constant is non-negative [17, 9].

We shall consider a toy model with the simplest possible set of ingredients, where there is only one harmonic $(1,1)$ form and no five-branes. We hope to clear up some issues, such as the contribution which the warping of the metric makes in flux contributions to the superpotential. We are not aiming to present a fully phenomenologically accurate description of low energy particle physics.

Before proceding, we shall review some of the ingredients of the improved version of low-energy heterotic M-theory described in ref. [6]. The theory is formulated on a manifold $\mathcal{M}$ with a boundary consisting of two disconnected components $\mathcal{M}_1$ and $\mathcal{M}_2$ with identical topology. The eleven-dimensional part of the action is conventional for supergravity,

$$S_{\mathrm{SG}} = \frac{1}{2\kappa_{11}^2}\int_{\mathcal{M}}\Bigg(- R(\Omega) - \bar{\Psi}_I\Gamma^{IJK}D_J(\Omega^*)\Psi_K - \frac{1}{24}G_{IJKL}G^{IJKL} - \frac{\sqrt{2}}{96}\left(\bar{\Psi}_I\Gamma^{IJKLMP}\Psi_P + 12\bar{\Psi}^J\Gamma^{KL}\Psi^M\right)G^*_{JKLM} - \frac{2\sqrt{2}}{11!}\epsilon^{I_1\ldots I_{11}}(C\wedge G\wedge G)_{I_1\ldots I_{11}}\Bigg)dv, \tag{1.1}$$

where G is the abelian field strength and Ω is the tetrad connection. The combination $G^* = (G+\hat{G})/2$, where hats denote the standardised subtraction of gravitino terms to make a supercovariant expression.

The boundary terms which make the action supersymmetric are,

$$S_0 = \frac{1}{\kappa_{11}^2}\int_{\partial\mathcal{M}}\left(K \mp \frac{1}{4}\bar{\Psi}_A\Gamma^{AB}\Psi_B + \frac{1}{2}\bar{\Psi}_A\Gamma^A\Psi_N\right)dv, \tag{1.2}$$

where K is the extrinsic curvature of the boundary. We shall take the upper sign on the boundary component $\partial\mathcal{M}_1$ and the lower sign on the boundary component $\partial\mathcal{M}_2$.

There are additional boundary terms with Yang-Mills multiplets, scaled by a parameter ϵ,

$$S_1 = -\frac{\epsilon}{\kappa_{11}^2}\int_{\partial\mathcal{M}}dv\left(\frac{1}{4}\left(\mathrm{tr}F^2 - \frac{1}{2}\mathrm{tr}R^2\right) + \frac{1}{2}\mathrm{tr}\bar{\chi}\Gamma^A D_A(\hat{\Omega}^{**})\chi + \frac{1}{4}\bar{\Psi}_A\Gamma^{BC}\Gamma^A\mathrm{tr}F^*_{BC}\chi\right), \tag{1.3}$$

where $F^* = (F + \hat{F})/2$ and $\Omega^{**} = (\Omega + \Omega^*)/2$. The original formulation of Horava and Witten contained an extra '$\chi\chi\chi\Psi$' term, but it is not present in the new version. The formulation given in ref. [7] was only valid to order R, and the extension of the theory to include the R^2 term has been reported recently [21].

The specification of the theory is completed by boundary conditions. For the tangential anti-symmetric tensor components,

$$C_{ABC} = \mp\frac{\sqrt{2}}{12}\epsilon\left(\omega^Y_{ABC} - \frac{1}{2}\omega^L_{ABC}\right) \mp \frac{\sqrt{2}}{48}\epsilon\,\mathrm{tr}\bar{\chi}\Gamma_{ABC}\chi. \tag{1.4}$$

where ω^Y and ω^L are the Yang-Mills and Lorentz chern-simons forms. These boundary conditions replace the modified Bianchi identity in the old formulation. A suggestion along these lines was made in the original paper of Horava and Witten [2]. For the gravitino,

$$\Gamma^{AB}\left(P_\pm + \epsilon\Gamma P_\mp\right)\Psi_A = \epsilon\left(J_Y{}^A - \frac{1}{2}J_L{}^A\right), \tag{1.5}$$

where $P_\pm$ are chiral projectors using the outwart-going normals and

$$\Gamma = \frac{1}{96}\mathrm{tr}(\bar{\chi}\Gamma_{ABC}\chi)\Gamma^{ABC}. \tag{1.6}$$

J_Y is the Yang-Mills supercurrent and J_L is a gravitino analogue of the Yang-Mills supercurrent.

The resulting theory is supersymmetric *to all orders* in the parameter ϵ when working to order R^2. The gauge, gravity and supergravity anomalies vanish if

$$\epsilon = \frac{1}{4\pi}\left(\frac{\kappa_{11}}{4\pi}\right)^{2/3}. \tag{1.7}$$

Further details of the anomaly cancellation, and additional Green-Schwarz terms, can be found in ref. [7].

2. Background

The reduction to four dimensions begins with a family of solutions to the field equations which are homogeneous in four dimensions. In this section we shall adapt the background-field solutions used orginally by Witten [3], (further developed in [22, 24, 23]), to the new formulation of low-energy heterotic M-theory. These solutions were obtained order by order in ϵ. We expand the solutions in a similar way, but starting from an action which is valid to all orders in ϵ gives better control of the error terms.

The ansatz for the background metric is based on a warped product $M \times S^1/Z_2 \times Y$ where Y is a Calabi-Yau space. In this metric there are two copies of the 4$-$dimensional manifold M, M_1 and M_2, separated by a distance l_{11}. Ideally, a typical value for the inverse radius of the Calabi-Yau space would be of order the Grand Unification scale 10^{16}GeV and the inverse separation would be of order 10^{14}GeV.

The explicit form of the background metric ansatz which we shall use is

$$ds^2 = V^{-2/3}dz^2 + V^{-1/3}\eta_{\mu\nu}dx^\mu dx^\nu + V^{1/3}(\tilde{g}_{a\bar{b}}dx^a dx^{\bar{b}} + \tilde{g}_{\bar{a}b}dx^{\bar{a}}dx^b). \tag{2.1}$$

where $\eta_{\mu\nu}$ is the Minkowski metric on M, $\tilde{g}_{a\bar{b}}$ a fixed metric on Y and $V \equiv V(z)$, $z_1 \le z \le z_2$. Our background metric ansatz is similar to one used by Curio and Krause [25], except that we use a different coordinate z in the S_1/Z_2 direction.

For simplicity, we shall restrict the class of Calabi-Yau spaces to those with only one harmonic $(1,1)$ form with components $i\tilde{g}_{a\bar{b}}$. In this case the background flux for the antisymmetrc tensor field depends on only one parameter α,

$$G_{ab\bar{c}\bar{d}} = \frac{1}{3}\alpha\left(\tilde{g}_{a\bar{c}}\tilde{g}_{b\bar{d}} - \tilde{g}_{a\bar{d}}\tilde{g}_{b\bar{c}}\right) \tag{2.2}$$

This ansatz solves the field equation $\nabla \cdot G = 0$. The exterior derivative of the boundary condition (1.4) implies

$$G = \begin{cases} -\frac{\epsilon}{\sqrt{2}}\left(\mathrm{tr}(F\wedge F) - \frac{1}{2}\mathrm{tr}(R\wedge R)\right) & \text{on } M_1 \\ +\frac{\epsilon}{\sqrt{2}}\left(\mathrm{tr}(F\wedge F) - \frac{1}{2}\mathrm{tr}(R\wedge R)\right) & \text{on } M_2 \end{cases} \tag{2.3}$$

Since $\mathrm{tr}(R\wedge R)$ is a $(2,2)$ form, it takes a similar tensorial form to the flux term in eq. (2.2). The boundary conditions are satisfied if $\mathrm{tr}(F\wedge F) = \mathrm{tr}(R\wedge R)$ on the visible brane M_1 and $F = 0$ and on the hidden brane M_2. The value of α can be related through eq. (2.3) to an integer β characterising the Pontrjagin class of the Calabi-Yau space [24],

$$\alpha = \frac{4\sqrt{2}\pi^2}{v_{\mathrm{CY}}^{2/3}}\epsilon\beta \tag{2.4}$$

where v_{CY} is the volume of the Calabi-Yau space.

The volume function $V(z)$ is determined by the exact solution of the 'zz' component of the Einstein equations,[1]

$$V(z) = 1 - \sqrt{2}\alpha z. \tag{2.5}$$

The metric ansatz is consistent with all of the Einstein equations apart from the ones with components along the Calabi-Yau direction, where the Einstein tensor vanishes but the stress energy tensor is $O(\alpha^2)$. The difference between an exact solution to the Einstein equations and the metric ansatz is $\delta g_{IJ} = O(\alpha^2)$. If we calculate the action to reduce the theory to four dimensions, then the error in the action is $O(\alpha^4)$. As long as we work within this level of approximation we can use the Calabi-Yau approximation as the background for our reduced theory. Note that this approximation is uniform in z, and having small values of α does not necessarily mean small warping.

We shall also need to know the background solutions for a Rarita-Schwinger field when it takes non-zero constant values on the boundary. It is convenient to redefine the Rarita-Schwinger field first by taking

$$\lambda_I = \Psi_I - \frac{1}{2}\Gamma_I\Gamma^J\Psi_J \tag{2.6}$$

[1]Our solution for V is equivalent to the one used by Lukas et al. in ref. [24] when adapted to our coordinate system. They express the solution as $V = b_0 H^3$. It is also equivalent to the background used by Curio and Krause in ref. [25], $V = (1 - \mathcal{S}_1 x^{11})^2$, when their $\mathcal{S}_1 = \alpha V_1^{-2/3}/\sqrt{2}$.

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The Rarita-Schwinger equation for λ_I becomes

$$\left(\Gamma^I D_I - \frac{\sqrt{2}}{96}\Gamma^{IJKL}G_{IJKL}\right)\lambda_P + \frac{\sqrt{2}}{4}G_{PJKL}\Gamma^{JK}\lambda^L = 0 \tag{2.7}$$

The solution with our metric/flux background and boundary conditions (1.5) is

$$\lambda_\mu = V^{1/12}\theta_\mu \otimes u_+ + V^{1/12}\theta^*_\nu \otimes u_- \tag{2.8}$$

where $u_\pm$ are the covariantly constant chiral spinors on the Calabi-Yau space and θ_μ is a chiral $4-$spinor.

3. Reduction

Reduction of low-energy heterotic $M-$theory to 5 or 4 dimensions follows a traditional route. The light fields in the $4-$dimensional theory correspond to the moduli of the background fields. We shall be focussing especially on the volume of the Calabi-Yau space and the separation of the two boundaries. These quantities can be expressed in terms of the values of V on the two boundaries, V_1 and V_2.

To allow for gravity in $4-$dimensions, the metric is replaced by

$$ds^2 = V^{-2/3}dz^2 + V^{-1/3}\Phi^2\tilde{g}_{\mu\nu}dx^\mu dx^\nu + V^{1/3}(\tilde{g}_{a\bar{b}}dx^a dx^{\bar{b}} + \tilde{g}_{\bar{a}b}dx^{\bar{a}}dx^b). \tag{3.1}$$

The factor Φ^2 is required to put the metric $\tilde{g}_{\mu\nu}$ into the Einstein frame,

$$\Phi = \left(V_1^{4/3} - V_2^{4/3}\right)^{-1/2}. \tag{3.2}$$

With this definition of the Einstein metric, the gravitational coupling in 4 dimensions is given by

$$\kappa_4^2 = \frac{2\sqrt{2}}{3}\frac{\kappa_{11}^2}{v_{\rm CY}}\alpha. \tag{3.3}$$

The reduction to the Einstein frame decouples the metric derivatives from V_1 and V_2. The kinetic terms for V_1 and V_2 become[2]

$$\frac{1}{2\kappa_4^2}\int_M \left(\frac{3}{2}(\partial^2_{V_1}\ln\Phi)(\partial_\mu V_1)(\partial^\mu V_1)\right. \tag{3.4}$$
$$\left. + \frac{3}{2}(\partial^2_{V_2}\ln\Phi)(\partial_\mu V_2)(\partial^\mu V_2) + 3(\partial_{V_1}\partial_{V_2}\ln\Phi)(\partial_\mu V_1)(\partial^\mu V_2)\right)d\tau$$

where τ is the volume element for the metric $\tilde{g}_{\mu\nu}$. Most authors like to introduce an extra length-scale ρ into the metric, which then appears in the definition of κ_4 [22, 24, 23]. However, this scale is a redundant variable and gives the false impression that the brane separation can be chosen arbitrarily.

[2]For a detailed discussion see [33]. The moduli fields used there are Q and R, related to our variables by $V_1^{2/3} = Q\cosh R$ and $V_2^{2/3} = Q\sinh R$. Note that their parameter $\alpha = \sqrt{3/2}$ for heterotic $M-$theory.

For the Yang-Mills multiplets we have to rescale the gaugino to normalise the kinetic terms correctly,

$$\chi_i = V^{1/4}\Phi^{-3/2}\tilde{\chi}_i \otimes u_+ + V^{1/4}\Phi^{-3/2}\tilde{\chi}_i^* \otimes u_-, \tag{3.5}$$

where $u_\pm$ are the covariantly constant chiral spinors on the Calabi-Yau space. The matter action for the hidden E_8 multiplet, for example, becomes

$$S_h = \frac{1}{4g^2}\int_M V_2 \left(\mathrm{tr}(F_2^2) + \tilde{\chi}_2\gamma^\mu D_\mu \tilde{\chi}_2\right) d\tau. \tag{3.6}$$

where the Yang-Mills coupling is

$$g^2 = \frac{\kappa_{11}^2}{2\epsilon v_{\mathrm{CY}}}. \tag{3.7}$$

Note that all of the model parameters κ_{11}, ϵ, α and v_{CY} can be expressed in terms of the integer β and measurable parameters κ_4 and g for the purposes of phenomenology.

The reduction from 5 dimensions to 4 has also been done using a superfield formalism by Correia et al [26]. This shows that in the $h_{1,1} = 1$ case the reduced theory is a supergravity model with V_1 and V_2 belonging to chiral superfields S_1 and S_2 with Kähler potential

$$K = -3\ln\left((S_1 + S_1^*)^{4/3} - (S_2 + S_2^*)^{4/3}\right) \tag{3.8}$$

Note that, for the real scalar components, the conformal factor introduced in eq. (2.1) and the Kähler potential are related by

$$\Phi = 2^{2/3}e^{K/6}, \tag{3.9}$$

making the action derived from the Kähler potential consistent with eq. (3.5). The superfields S_1 and S_2 also appear in the reduced theory as gauge kinetic functions for the E_8 Yang-Mills supermultiplets.

4. Condensates and fluxes

Fermion condensates and fluxes of antisymmetric tensor fields may both play a role in the stabilisation of moduli fields. In the context of low energy heterotic M-theory the most likely candidate for forming a fermion condensate is the gaugino on the hidden brane, since the effective gauge coupling on the hidden brane is larger and runs much more rapidly into a strong coupling regime than the gauge coupling on the visible brane.

In the new formulation of low energy heterotic M-theory, gaugino condensates act as sources for the field strength G through the boundary conditions. Other non-zero contributions to the field strength are possible, and we shall include some of these in the next section.

4.1 Gaugino condensate

The ansatz for a gaugino condensate on the boundary M_i is [18],

$$\bar{\chi}_i\Gamma_{abc}\chi_i = \Lambda_i\omega_{abc} \tag{4.1}$$

where Λ_i depends only on the modulus V_i and ω_{abc} is the covariantly constant 3−form on the Calabi-Yau space (i.e. the one with volume $v_{\rm CY}$). The gaugino condensate appears in the boundary conditions for the antisymmetric tensor field and induces non-vanishing components

$$C_{abc} = \frac{1}{6}\xi\omega_{abc}. \tag{4.2}$$

where ξ is a complex scalar field. The field strength associated with these tensor components is

$$G_{abcz} = -(\partial_z\xi)\omega_{abc}. \tag{4.3}$$

When we solve the field equation $\nabla \cdot G = 0$ with boundary conditions (1.4), we get

$$\xi = -\frac{\sqrt{2}}{8}\Lambda_1\epsilon\Phi^2\left(V^{4/3} - V_2^{4/3}\right) - \frac{\sqrt{2}}{8}\Lambda_2\epsilon\Phi^2\left(V^{4/3} - V_1^{4/3}\right). \tag{4.4}$$

where Φ was defined in eq. (3.2). The new flux contribution is

$$G_{abcz} = -\frac{\alpha}{3}(\Lambda_1 + \Lambda_2)\epsilon\Phi^2\omega_{abc}V^{1/3}. \tag{4.5}$$

The appearance of Φ in this formula is independent of the normalisation of the metric, but its presence here will prove essential for the consistency of the reduction.

The non-zero flux depends on V_1 and V_2 through the Φ term and through Λ, which depends on the volume factors V_1 and V_2 in the gaugino couplings. The G^2 term in the action reduces to a potential V_c in the Einstein frame, where

$$V_c = \frac{\sqrt{2}}{3}\frac{\epsilon^2 v_{\rm CY}}{2\kappa_{11}^2}\alpha\Phi^6|\Lambda_1 + \Lambda_2|^2. \tag{4.6}$$

This is not the whole story, however, and we shall attempt to find the potential by a better method, using a reduction of the fermion sector in section 4.3.

4.2 Fluxes

The boundary conditions on the flux G do not fully determine the value of G_{abcz}, and it is possible to have a non-zero flux of the form (4.5) even in the absence of a gaugino condensate. In this situation the main restriction is topological, related to the quantisation rule [28, 27],

$$\frac{1}{4\pi\epsilon}\int_{C_4}\frac{G}{2\pi} + \frac{1}{32\pi^2}\int_{C_4}\mathrm{tr}(R\wedge R) = n \tag{4.7}$$

where C_4 is any closed four-cycle. We have to apply this rule in the presence of boundaries, where there are modifications as suggested by Lukas et al. [19]. We shall use,

$$\frac{1}{4\pi\epsilon}\left(\int_{C_4} G - 6\int_{\partial C_4} C_{\partial C_4}\right) + \frac{1}{32\pi^2}\int_{C_4}\mathrm{tr}(R\wedge R) = n, \tag{4.8}$$

where $C_{\partial C_4}$ denotes the restriction of the antisymmetric tensor C to the boundary.

We modify our earlier ansatz so that

$$C_{abc} = \frac{1}{6}\xi\omega_{abc}, \tag{4.9}$$

$$C_{zab} = \frac{1}{12}(\partial_z\sigma)\, b_{ab}. \tag{4.10}$$

This ansatz introduces a non-trivial topological structure with locally defined antisymmetric tensor components b_{ab}, defined such that $db = \omega$.

Since the reduction ansatz does not produce a well-defined 3-form field, we have to look for a suitable context in which C can be defined. We shall regard C as a gerbe connection, as described in the appendix. This will give a framework which is consistent with the quantisation rule (4.7). Furthermore, the rules for combining gerbe connections given in the appendix require an ansatz of the form $d\sigma \wedge b$, and this explains why we have to use $\partial_z\sigma$ in (4.10), rather than an arbitrary 1−form field.

The solution to the field equation $\nabla \cdot G = 0$ is now

$$G_{abcz} = -\frac{\alpha}{3}(\Lambda_1 + \Lambda_2 + \lambda)\epsilon\Phi^2\omega_{abc}V^{1/3} \tag{4.11}$$

where λ depends on the values of σ on the boundaries M_1 and M_2,

$$\lambda = \frac{4\sqrt{2}}{\epsilon}(\sigma_1 - \sigma_2) \tag{4.12}$$

The λ term is the M-theory analogue of the flux term introduced by Dine et al. [18] in an attempt to cancell the gaugino condensate in type IIB superstring theory.

The value of λ can be determined by the quantisation rule (4.8) using $C_4 = S_1/Z_2 \times C_3$,

$$\lambda = \frac{32\sqrt{2}\pi^2}{c v_{\mathrm{CY}}^{1/2}} n, \qquad n = 0, 1, \ldots \tag{4.13}$$

where the constant c is defined by

$$c = \frac{1}{v_{\mathrm{CY}}^{1/2}} \int_{C_3} \omega. \tag{4.14}$$

This is the analogue of the flux quantisation in type IIB superstring theory [28].

We can also obtain the quantisation rule independently of eq. (4.8) by using the field σ and the product rules described in the appendix. From (4.12) and (A.5), the flux is given in the terms of the charge q_σ by

$$\lambda = \frac{4\sqrt{2}}{\epsilon}\frac{2\pi n}{q_\sigma} = \frac{4\sqrt{2}}{\epsilon}\frac{n q_b}{q_C}. \tag{4.15}$$

With the charge $q_b = 2\pi c^{-1} v_{\mathrm{CY}}^{-1/2}$ given in the appendix, and $q_C = (4\pi\epsilon)^{-1}$ from (4.7), this reproduces eq. (4.13)

4.3 Superpotential

The superpotential W for the moduli superfields S_1 and S_2 can be obtained from the gravitino mass terms

$$\mathcal{L}_{3/2\ \mathrm{mass}} = \frac{1}{2\kappa_4^2} e^{K/2} \left(W \bar{\theta}^\mu \theta_\mu + c.c \right) \tag{4.16}$$

provided that the θ_μ are correctly normalised,

$$\mathcal{L}_{3/2\ \mathrm{kinetic}} = \frac{1}{2\kappa_4^2} \left(\bar{\theta}^\mu \gamma^\nu D_\nu \theta_\mu + c.c \right). \tag{4.17}$$

Note that these are valid in the Einstein frame and indices are raised with the metric $\tilde{g}^{\mu\nu}$.

The 4−dimensional action for the gravitino field can be obtained using the background (2.8) and metric (3.1).[3] We introduce an additional factor $a \equiv a(V_1, V_2)$ to allow us to adjust the normalisation,

$$\lambda_\mu = a V^{1/12} \theta_\mu \otimes u_+ + a^* V^{1/12} \theta^*_\nu \otimes u_- \tag{4.18}$$

For the kinetic term,

$$\bar{\lambda}^\mu \Gamma^\nu D_\nu \lambda_\mu |g|^{1/2} = |a|^2 \Phi\, V^{2/3} \left(\bar{\theta}^\mu \gamma^\nu D_\nu \theta_\mu + c.c. \right) |\tilde{g}|^{1/2}. \tag{4.19}$$

Similarly, for the flux term,

$$\bar{\lambda}^\mu \Gamma^{abcz} G_{abcz} \lambda_\mu |g|^{1/2} = |a|^2 \Phi^2\, V^{1/3} \omega^{abc} G_{abcz} \left(\bar{\theta}^\mu \theta_\mu + c.c. \right) |\tilde{g}|^{1/2}. \tag{4.20}$$

These terms have to be integrated over the Calabi-Yau space and over the z coordinate. However, when we substitute the flux from eq. (4.11), both terms have an identical dependence on z. Therefore the superpotential can be read off directly from eqs. (4.19) and (4.20) by comparison with eqs. (4.17) and (4.16). The normalisation factor Φ cancells due to eq. (3.9), and we get

$$W = -3\sqrt{2} \alpha \epsilon \left(\Lambda_1 + \Lambda_2 + \lambda \right) \tag{4.21}$$

where Λ_1 and Λ_2 are the amplitudes of the condensates (4.1) and λ is the quantised flux.

Most discussions of the condensate induced superpotential do not take the warping of the metric into account. We have found that the warping of the metric background has had no effect on the superpotential. Krause [29] also finds that the warping does not affect the condensate contribution to the superpotential, but he claims a warping dependence in the flux term. This can be traced to a formula for the superpotential given by Anguelova and Zoubos [30]. We can derive a similar formula by integrating eq. (4.20) and comparing to eq. (4.16),

$$W = \frac{4\alpha}{v_{\mathrm{CY}}} \frac{|a|^2}{\Phi} \int_{Y \times [z_1, z_2]} V^{-1/6} G \wedge V^{1/2} \bar{\omega}. \tag{4.22}$$

Note that a depends on the moduli fields when we normalise the gravitino kinetic term using (4.19). The result derived by Anguelova and Zoubos does not contain the factor $|a|^2/\Phi$.[4]

[3] The condensate will change the gravitino background to order $\alpha\Lambda$ and the action to order $(\alpha\Lambda)^2$. We are therefore entitled to ignore this correction to the background when calculating the superpotential.

[4] If we use their gravitino field $\propto V^{-1/12}$ rather than our solution $\propto V^{1/12}$, the factor disappears but the $V^{-1/6}$ inside the integral becomes $V^{-1/2}$. The final superpotential obtained after integration is unchanged.

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5. Moduli stabilisation

Moduli stabilisation can be achieved by following a similar pattern to moduli stabilisation in type IIB string theory [17]. The first stage involves finding a suitable superpotential which fixes the moduli but leads to an Anti-de Sitter vacuum. The negative energy of the vacuum state is then raised by adding a non-supersymmetric contribution to the energy.

The potential is given in terms of the Kähler potential K and the superpotential W,

$$V = \kappa_4^{-2} e^K \left(g^{i\bar{j}} (D_i W)(D_j W)^* - 3WW^* \right), \tag{5.1}$$

where $g_{i\bar{j}}$ is the hessian of K and

$$D_i W = e^{-K} \partial_{V_i} (e^K \, W). \tag{5.2}$$

Minima of the potential occur when $D_i W = 0$. If these minima exist, their location is fixed under supersymmetry transformations. However, the boundary conditions at the potential minima are not generally preserved by supersymmetry and the theory at a supersymmetric minimum is not necessarily supersymmetric in higher dimensions. This distinction is subtle, but important because it allows for mechanisms which produce de Sitter minima.

We shall examine the supersymmetric minima of the potential for two toy models. We shall concentrate on general features rather than obtaining a precise fit with particle phenomenology.

5.1 Double-condensate

Following the type IIB route, we assume the existence of a flux term W_f in the superpotential which stabilises the $(2,1)$ moduli, and then remains largely inert whilst the other moduli are stabilised.

The gauge coupling on the hidden brane runs to large values at moderate energies and this is usually taken to be indicative of the formation of a gaugino condensate. Local supersymmetry restricts the form of this condensate to [31]

$$\Lambda_2 = B_2 \, v_{\rm CY}^{-1/2} e^{-\mu V_2} \tag{5.3}$$

where B_2 is a constant and μ is related to the renormalisation group β-function by

$$\mu = \frac{6\pi}{b_0 \alpha_{\rm GUT}}, \quad \beta(g) = -\frac{b_0}{16\pi^2} g^3 + \dots. \tag{5.4}$$

The gauge coupling on the visible brane is supposed to run to large values only at low energies to solve the hierarchy problem, and a low energy condensate would have a negligible effect on moduli stabilisation. There might, however, be a separate gauge coupling from part of the E_6 symmetry on the visible brane which becomes large at moderate energies with a significant condensate term. The requirement for this to happen is a large β-function, possibly arising from charged scalar field contributions. The total superpotential for such a model would be given by combining eq. (4.21) with W_f,

$$W = b e^{-\mu V_2} + c e^{-\tau V_1} - w, \tag{5.5}$$

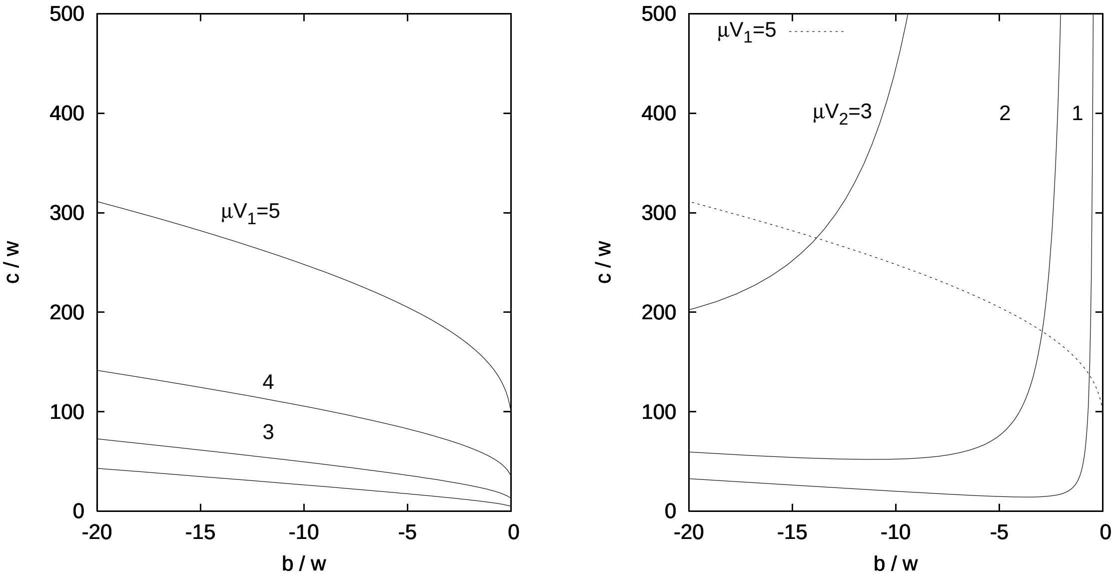

Figure 1: The values of the volume moduli at the minimum of the potential with two condensates and $\tau/\mu = 1.2$. The left panel shows values of V_1 and the right panel shows values of V_2.

where $w = -W_f$ and b, c are constants, which we assume to be real but not necessarily positive.

The fields at the minimum of the potential could be complex, and we therefore separate real and imaginary parts,

$$V_i = u_i + iv_i. \tag{5.6}$$

The superderivatives of the potential are

$$D_1W = -c\tau e^{-\tau V_1} - 2(u_1^{4/3} - u_2^{4/3})^{-1}u_1^{1/3}W, \tag{5.7}$$

$$D_2W = -b\mu e^{-\mu V_2} - 2(u_1^{4/3} + u_2^{4/3})^{-1}u_2^{1/3}W. \tag{5.8}$$

Solving for the values of V_1 and V_2 at the minimum of the potential is not very informative. Instead, we express the parameters b, c and w in terms of the values of V_1 and V_2 at the supersymmetric minimum,

$$\frac{b}{w} = \frac{-2u_2^{1/3}e^{\mu V_2}\mu^{-1}}{u_1^{4/3} - u_2^{4/3} - 2\mu^{-1}u_2^{1/3} + 2\tau^{-1}u_1^{1/3}}, \tag{5.9}$$

$$\frac{c}{w} = \frac{2u_1^{1/3}e^{\tau V_1}\tau^{-1}}{u_1^{4/3} - u_2^{4/3} - 2\mu^{-1}u_2^{1/3} + 2\tau^{-1}u_1^{1/3}}. \tag{5.10}$$

We conclude from these expressions that, if b/w and c/w are real, then V_1 and V_2 are both real. (If b and c are not real, then it becomes difficult to satisfy the background field equations on the antisymmetric tensor field with the resulting complex boundary conditions).

Supersymmetric minima exist for $b < 0$ and $c > 0$. The values of V_1 and V_2 at the minima are shown in figure 1. At the minima of the potential, the flux term $|W_f|$ is larger than the gaugino condensate terms. This is consistent with the idea that we consider the stabilisation of the $(2, 1)$ moduli independently of the other moduli.

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5.2 Other non-perturbative terms

If there are no high energy condensates on the visible brane, then we can replace the condensate on the visible brane with another non-perturbative effect. The usual candidate for this is a membrane which stretches between the two boundaries. The area of the membrane $\propto V_1 - V_2$ and the type of contribution this gives to the superpotential is

$$W_{np} = ce^{-\tau(V_1 - V_2)}. \tag{5.11}$$

The total superpotential for the toy model is given by

$$W = be^{-\mu V_2} + ce^{-\tau(V_1 - V_2)} - w, \tag{5.12}$$

where $w = -W_f$ and b, c are constants. .

This time the parameters b, c and w given in terms of the values of V_1 and V_2 at the supersymmetric minimum are

$$\frac{b}{w} = \frac{-2u_2^{1/3} e^{\mu V_2} \mu^{-1}}{u_1^{4/3} - u_2^{4/3} + 2\mu^{-1}(u_1^{1/3} - u_2^{1/3}) + 2\tau^{-1} u_1^{1/3}}, \tag{5.13}$$

$$\frac{c}{w} = \frac{2u_1^{1/3} e^{\tau(V_1 - V_2)} \tau^{-1}}{u_1^{4/3} - u_2^{4/3} + 2\mu^{-1}(u_1^{1/3} - u_2^{1/3}) + 2\tau^{-1} u_1^{1/3}}. \tag{5.14}$$

Again we conclude from these expressions that V_1 and V_2 are both real. The values of the moduli at the supersymmetric minima of the potential are shown in figure 2, where we have taken $\tau = \mu$. Other values of τ give a qualitatively similar figure. There are always supersymmetric minima in the parameter region indicated on the figure.

6. Conclusion

We have started from an improved formulation of low energy heterotic M-theory and revisited some aspects of gaugino condensation and moduli stabilisation. Many of the undesirable features that have been introduced previously, such as hiding away delta functions in field redefinitions and modifying the Bianchi identities have now become unnecessary. The reduction of low energy heterotic M-theory to four dimensions also displays some new features. The non-trivial topology of the anti-symmetric tensor field shows up very clearly, for example.

Having an action which is valid to all orders in the gravitational coupling means that now the warping of the five-dimensional metric can be taken into account consistently. We would like to stress that a small brane charge α does not necessarily imply small warping.

The final superpotential contains no surprises. It takes the standard form expected for a gaugino condensate in any supersymmetric theory [31], and both condensate and quantised flux contributions to the superpotential contain no corrections due to the warping of the metric in higher dimensions.

It remains to be seen how the other ingredients of low energy heterotic M-theory which we have neglected in this paper enter into the mix, for example the extra $(1, 1)$ moduli,

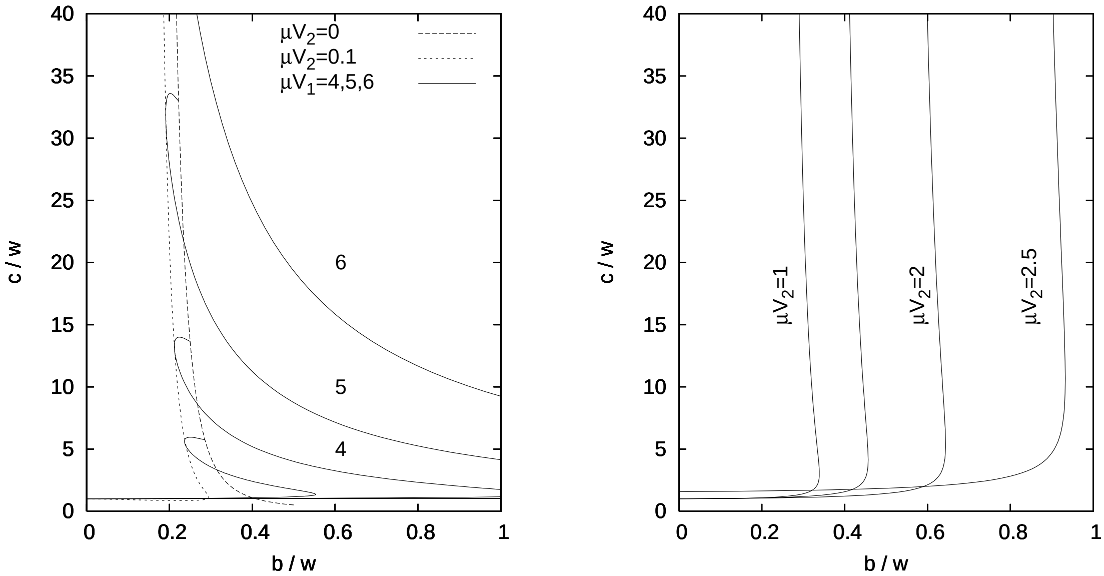

Figure 2: The values of the volume moduli at the minima of the potential with W_{np} and $\mu = \tau$. The left panel shows values of V_1 and the right panel shows values of V_2. There are no solutions to the left of the leftmost dashed line ($\mu V_2 = 0.1$) and one solution to the right of the rightmost dashed line. The strip between the dashed lines indicates a parameter region with small values of V_2.

five-branes and anti five-branes may all play a role in a realistic model [11]. Some features of the present calculation may be helpful in these generalisations. For example, the five dimensional superfield formalism gives a good guide as to good choices of moduli fields, in our case these were the Calabi-Yau volumes rather than the brane separation. The inclusion of five-branes in the improved formalism for heterotic M-theory still remains to be developed.

We have not made full use in this paper of the fermion boundary conditions (see eq. (1.5)). If the theory is reduced to five dimensions in the presence of a gaugino condensate on the hidden brane, then the boundary conditions break the five-dimensional supersymmetry. The four dimensional theory will then consist of the light modes, described by the superpotential W, and a non-zero quantum vacuum energy from all of the other modes. We are presently considering situations where this vacuum energy can raise the energy at the minimum of the potential to positive values.

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A. Gerbe connections

In this appendix we give a simplified description of connections on gerbes, following the review by Hitchin [32]. We shall define a 1−gerbe with respect to a given open cover U_α of a manifold. We use the notation $U_{\alpha\beta\ldots\gamma}$ to refer to the intersection of the sets $U_\alpha \ldots U_\gamma$. The same subscripts on a tensor indicate that the tensor is only defined on the corresponding region.

A connection with charge q on a 1−gerbe is defined to be a set

$$\{B_\alpha, A_{\alpha\beta}, g_{\alpha\beta\gamma}\} \tag{A.1}$$

where the B_α are 2−forms, $A_{\alpha\beta}$ are 1−forms and $g_{\alpha\beta\gamma}$ are complex numbers of unit modulus. The values depend on the ordering of the indices, for example $g_{\alpha\beta\gamma} = g_{\gamma\alpha\beta} = g^{-1}_{\beta\alpha\gamma}$. The forms are related by a set of consistency relations,

$$B_\alpha - B_\beta = -dA_{\alpha\beta} \tag{A.2}$$

$$A_{\alpha\beta} + A_{\beta\gamma} + A_{\gamma\alpha} = -i\, q^{-1}\, d\ln\, g_{\alpha\beta\gamma} \tag{A.3}$$

$$g_{\alpha\beta\gamma}g_{\gamma\beta\delta}g_{\beta\gamma\delta}g_{\alpha\gamma\delta} = 1. \tag{A.4}$$

A simple generalisation allows the definition of p−gerbes with connection for $p = 0, 1\ldots$, the leading terms being $(p+1)$−forms. The case 0−gerbe with connection is equivalent to a U(1) fibre bundle with connection.

The curvature of the p−gerbe with connection is defined to be dB_α and it is independent of the choice of open set α. It defines an integral class, i.e.

$$\int_{C_{p+2}} \frac{dB}{2\pi} = \frac{n}{q}, \tag{A.5}$$

where n is an integer and C_{p+2} is a closed cycle. This generalises the Dirac quantisation condition to p−forms. The converse also holds, i.e. given a closed $p+2$ form and the quantisation condition, then there exists a gerbe and connection. In the case of supergravity, it is the existence of a quantisation condition for the antisymmetric tensor flux which indicates that the antisymmetric field should be associated with a gerbe.

The wedge product of two connections on a gerbe, lets say B and C, defined by taking the wedge products of the form components term by term, usually fails to satisfy the consistency relations. However, with a little care, it is possible to define $B \wedge dC$ and $C \wedge dB$ so that these are gerbe connections. If B is a connection on a p_1−gerbe with charge q_1 and C is a connection on a p_2−gerbe with charge q_2, then $B \wedge dC$ and $C \wedge dB$ are connections on a $p_1 + p_2 + 1$−gerbe with charge $q_1q_2/2\pi$.

We can apply the gerbe technology to the Calabi-Yau 3−form ω. Introduce a closed cycle C_3 and define a constant c by

$$\int_{C_3} \omega = c\, v_{\rm CY}^{1/2} \tag{A.6}$$

There is only one 3-cycle on the Calabi-Yau space which gives a non-vanishing result, and the associated quantisation condition implies there exists a gerbe connection b with curvature $db = \omega$ and charge $q_b = 2\pi c^{-1} v_{\rm CY}^{-1/2}$.

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